

9fZcfgW\ib[ibX6Y_\AadZib[XYgGh][a UgdgmW\]gW\Yf9f_fUb_ib[Yb.8Ug9b[U[YaYbhXYg8ND;



8]Y bY[Uh]j Yb 5i gk]f_i b[Yb j cb Gh][a Uh]g]Yfi b['Ui Z'X]Y dgmWX]gWXY ; Ygi bX\Y]h

Gh][a Uh]g]Yfi b['j cb'A YbgW\Yb'a]h'dgmW\]gW\Yb'9f_fUb_i b[Yb $Z\emptyset\fh'ni$ '5i g[fYbni b['i bX'6YbUW\hY]`][i b["'8]Y': c`[Yb' $Z\emptyset$ f'X]Y

 $6YhfcZYbYb'_ObbYb'gWk Yfk]Y[YbX'gY]b"'DfcZYggcf'8f"'B]Wc`UgF @gWk Z'Y]b'fYbca a]YfhYf'Gh][a UZcfgWk Yf'Ub'XYf'I b]j Yfg]h\[Ah'I `a \(\frac{7}{2} \) Yf_`\[Ah'I \\ K Ybb'A YbgWk Yb'a]h'Y]bYf'dgmWk]gWk Yb'9f_fUb_i b[gh][a Uh]g]Yfh'k YfXYb\[Ar'I \\ YfUhYb'g]Y'cZh']b'Y]bYb'HYi ZY`g_fY]g'Ui gF @W_ni [\(\frac{7}{2} \) 8 \Ua CfU`]g]Yfi b['i bX'gc[Uf'Y]bYa 'Yf\O\hYb Gi]n]Xf]g]_c" \(8 \Ua \) 8 \Ua \(8 \Ua \) 8 \Yi \hgwk Y' \NYbhfi a 'Z\@f'DgmWk]gWk Y'; Ygi bX\Y]h \(f8 \ND; \(\frac{1}{2} \) \Uh'Yg'g]Wk'ni a '\N]Y`'[YgYhnh\[Ar'I \) Gh][a U'ni 'YfZcfgWk Yb'i bX \\ ni 'fYXi n]YfYb"'9]b'VYgcbXYfYg'Dfc'Y_h'Ua 'GhUbXcfh'A Ubb\Y]a! \\ <Y]XY`VYf[!I `a ']gh' \(\Da' \) K \(@fXY'ni 'g]Wk'ghY\Yb \(\frac{7}{2} \) XUg'6Yhfc\(ZZYbYb'\) \(XUVY]'\]`'\]'\[\frac{7}{2} \) Thz'c\(ZZYb'\) \(\@VYf']\\ fY'? fUb_\Y]h'ni '_ca a i b]n]YfYb"'$

Gh][a Uh]q]Yfi b['VUq]Yfh'XUfUi Zž'XUqq'Y]bY'DYfqcb'b]W\h'U`q =bX]|]Xi i a 'VYhfUW\hYh'k]fXž'gcbXYfb 'Ub\UbX'| cb 'A Yf_a U`Ybž'X]Y]\fYf'; fi ddY'ni [YgWXf]YVYb'k YfXYb"'Hmd]gWXY'Jcfi fhY]'Y'g]bX VY]gd]Y`gk Y]gY. A YbgW\Yb a]h'dgmW\]gW\Yb 9f_fUb_i b[Yb g]bX i b ZÀ \][ž [Y ZÀ \ f`]W\ 'c X Y f`g Y `V gh`g W\ i `X" = b `X Y f`: U W\ k Y `h VYnY]WXbYh'a Ub'X]Y'6Y`Ughi b['Xi fWX'Gh][a Uh]g]Yfi b['U`g' nk Y]hY $? fUb_\Y]h \ \check{z} X U g]Y c Zh VY UghY b XY f]gh U g X]Y 9f_fUb_i b [gY Vgh"]$ G]Y Ubb X]Y @YVYbgei U`]hÀh'Yf\YV`]W VYY]bhfÀW h][Ybž']bXYa 'g]Y ni 'gcn]U`Yf'=gc`Uh]cb'i bX'gW\`YW\hYfYf'; Ygi bX\Y]h'ZØ\fh'i bX $gc[UfY]bYK]f_gUaY6YVbXib[iYfV]bXYfh"J]YY6YhfcZZYbY$]bhYfbU`]q]YfYb'X]YgY'JcfifhY]`Yž'kUg'U`g'GY`Vghgh][a Uh]q]Yfib[k U\fqWXY]b`]WX i bZÀ\][" '8]Y'DfcV`Ya Uh]_']gh'k Y]h'j YfVfY]hYh '9]bY k Y`hk Y]hY`Ghi X]Y`Yf[UVž`XUgg`Yhk U`, \$`DfcnYbh`XYf`VYZfU[hYb A YbgWX Yb a]h GWX |ncd\fYb]Y i bX 8 YdfYgg|cb Gh][a Uh]g]Yfi b[Yf`YVh'\UVYb"

Gh][a Uh]g]Yfi b['VYhf]ZZh'b]W\h'bi f'X]Y'6YhfcZZYbYb'gY'Vghž'gcbXYfb
Ui W\ XYfYb'5b[Y\Of][Y"'<Y]_Y'DYhYfY]h!N]dZY`'j ca
6i bXYgj YfVUbX'XYf'5b[Y\Of][Yb'dgmW\]gW\ Yf_fUb_hYf
A YbgW\Yb'f\b5d?\L'gU[h'5i W\ 5b[Y\Of][Ya UW\Yb']a a Yf'bcW\
gW\a Yfn\UZhY'9fZU\fi b[Yb'a]h'Gh][a Uh]g]Yfi b[']a 'h\a [`]W\Yb
@YVYb"'\L\gVYgcbXYfY'Yb[Y'5b[Y\Of][Y'gW\k Yf'dgmW\]gW\
Yf_fUb_hYf'A YbgW\Yb'\UVYb'Y]b'Yf\O\hYg'F]g]_cz'gY`Vgh
dgmW\]gW\Y'; Ygi bX\Y]hgdfcV`Ya Y'ni 'Ybhk]W_Y`b"

 $8 \ Ug`8 \ ND; \ `\ Uh`Yg`g]W\ `ni f`5i \ Z[\ UVY`[\ Ya \ UW hž`b]W\ h`bi f`X]Y$

5i Z_`Àfi b[g_Ua dU[bYbžX]Y`\DZcfa Uh]cbYb`ØVYf`dgmW\]gW\Y
9f_fUb_i b[Yb'j Yfa]hhY`bž`\UVYb'g]W\'U`g'k Yb]['YZZY_h]j ']a
5VVUi 'j cb'Gh][a U'Yfk]YgYb"'GhUhhXYggYb'\Uh'g]W\'[YnY][hž'XUgg
Dfc[fUa a Yž'X]Y'XYb'X]fY_hYb'gcn]U`Yb'?cbhU_h'nk]gW\Yb
A YbgW\Yb'a]h'i bX'c\bY'dgmW\]gW\Y'9f_fUb_i b[Yb'ZÒfXYfbž
k]f_gUa Yf'g]bX"'Gc`W\Y'Dfc[fUa a Y'_ObbhYb'VY]gd]Y`gk Y]gY']b
I bhYfbY\a Ybž'XYf'Dc`]nY]'cXYf']b'?`]b]_Yb'Y]b[YgYhnh'k YfXYbž'i a
Gh][a Uh]g]Yfi b['ni 'VY_Aa dZYb"

9]b 6Y]gd]Y`ZØfY]bYb_cbhU_hcf]Ybh]YfhYb 5bgUhn]ghXUg Dfc[fUa a 65GH5 Ub GWXi Yb Ua 8 ND; !GhUbXcfhA ØbWXYb! 5i [gVi f["6Y]?cbhU_hgYa]bUfYb`\UVYb GWXØ`Yf]bbYb`i bX $GWXØ`YfX]Y`A O[`]WX_Y]h\check{z}'g]WX a]hA]h[`]YXYfb XYf <math>\Rightarrow$]h]Uh]j Y Ui gni hUi gWXYb \check{z}' j cb XYbYbY']bYf ØVYf gY]b @YVYb a]hY]bYf dgmWX]gWXYb 9f_fUb_i b[VYf]WXhYhi bX j cb Y]bYa 6Y\UbXY`bXYb VY[`Y]hYh'k]fX"9]bY k]ggYbgWX UZh`]WXY 6Yk Yfhi b['gc```_`ÀfYb \check{z} 'cV Y]b 'gc`WXYg Dfc[fUa a 'J cfi fhY]`Y`VY]`GWXØ`Yfb UVVUi Yb'_Ubb" 8]Yg'_ObbhY`Ui WX`X]Y`K Y]hYfYbhk]W_`i b['XYg'Dfc[fUa a g'ZØf UbXYfY'6Yj O`_Yfi b[g[fi ddYb`Yfa O[`]WXYb"

 $8 \ Ug`8 \ ND; \ gYhnhig]W\ gY]h`A \ U]`\&\$\&' \ `XUZØf`Y]b\check{z}`X]Y`dgmW\&]gW\&Y\\; \ Ygi \ bX\Y]h`XYf`6Yj \ O`_Yfi \ b[`ni 'j YfVYggYfb'i bX`dgmW&]gW&Y\\ 9f_fUb_i \ b[Yb'ni 'Ybhgh][a \ Uh]g]YfYb'' 5b'gYW&g`GhUbXcfhYb']b\\ 8 \ Yi \ hgW\&`UbX`ZcfgW&Yb'9I \ dYfhYb'[Ya Y]bgUa 'a]h`6YhfcZZYbYbž\\ 5b[Y\Of][Yb'i \ bX`]bhYfbUh]cbU`Yb'K]ggYbgW&UZh`Yfbži a 'X]YgYg\\ N]Y``ni 'YffY]W&Yb''I \ bhYf`k k k "Xnd["cf['Z]bXYb' <math>\Rightarrow$ hYfYgg]YfhY k Y]hYfY' \Rightarrow Zcfa Uh]cbYb'ni 'Cf[Ub]gUh]cbž: cfgW&i b[gdfc Y_hYb'i bX

 $\label{lem:normalize} $$N]Y`Yb`gck]Y`\]`ZfY]WXY`HYI hY`i bX`@]b_g`ni a `H\Ya U`dgmWX]gWXY ; Ygi bX\Y]h"$

 $K \]ggYbgWX \ UZh`]WX \ Yf'5bgdfYWX \ dUfhbYf.$

DfYggY_cbhU_h

A 5GH9FA 98 \pm 5; a V <

8f".7cfXi `U.6Ui a g

HY`". '\$%) %'+\$%&) , ' -

dfYggY4 Xnd["cf[

kkk"Xnd["cf[

8 Y hU]`(

6Ygi WXYb G]Y i bg Ui Z. a Y]b!VYf `]b "bYh